Internal Model Control: A Comprehensive View

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1 Internal Model Control Structure - (IMC)

Internal Model Control (IMC) forms the basis for the systematic control system design methodology that is the primary focus of this text. The first issue one needs to understand regarding IMC is the IMC *structure* (to be distinguished from the IMC design procedure).

Figure 1B is the "Internal Model Control" or "Q-parametrization" structure. The IMC structure and the classical feedback structure (Figure 1A) are equivalent representations; Figure 2 demonstrates the evolution of the IMC structure. We will show that the design of $q(s)$ is more straightforward and intuitive than the design of $c(s)$. Having designed $q(s)$, its equivalent classical feedback controller $c(s)$ can be readily obtained via algebraic transformations, and vice-versa

$$c = \frac{q}{1-\tilde{p}q} \tag{1}$$

$$q = \frac{c}{1+\tilde{p}c} \tag{2}$$

1.1 Closed-loop transfer functions, IMC structure

A statement of the sensitivity ϵ and complementary sensitivity η in terms of the internal model $\tilde{p}$ and controller $q(s)$ corresponds to:

$$y = \frac{pq}{1+q(p-\tilde{p})}r + \frac{1-\tilde{p}q}{1+q(p-\tilde{p})}d \tag{3}$$

$$= \eta(s)r(s) + \epsilon(s)d(s) \tag{4}$$

In the absence of plant/model mismatch ($p = \tilde{p}$), these functions simplify to

$$\tilde{\eta}(s) = \tilde{p}q \qquad \tilde{\epsilon}(s) = 1 - \tilde{\eta}(s) = 1 - \tilde{p}q \qquad \tilde{p}^{-1}\tilde{\eta} = q \tag{5}$$

which lead to the following expressions for the input/output relationships between y, u, e and r, d, and n:

$$y = \tilde{p}qr + (1-\tilde{p}q)d - \tilde{p}qn \tag{6}$$

$$u = qr - qd - qn \tag{7}$$

$$e = (1-\tilde{p}q)r - (1-\tilde{p}q)d - (1-\tilde{p}q)n \tag{8}$$

1.2 Internal Stability

1. Assume a perfect model ($p = \tilde{p}$). The IMC system (Figure 1B) is internally stable (IS) if and only if both p and q are stable.

2. Assume that p is stable and $p = \tilde{p}$. Then the classical feedback system (Figure 1A) with controller according to Equation (1) is IS if and only if q is stable.

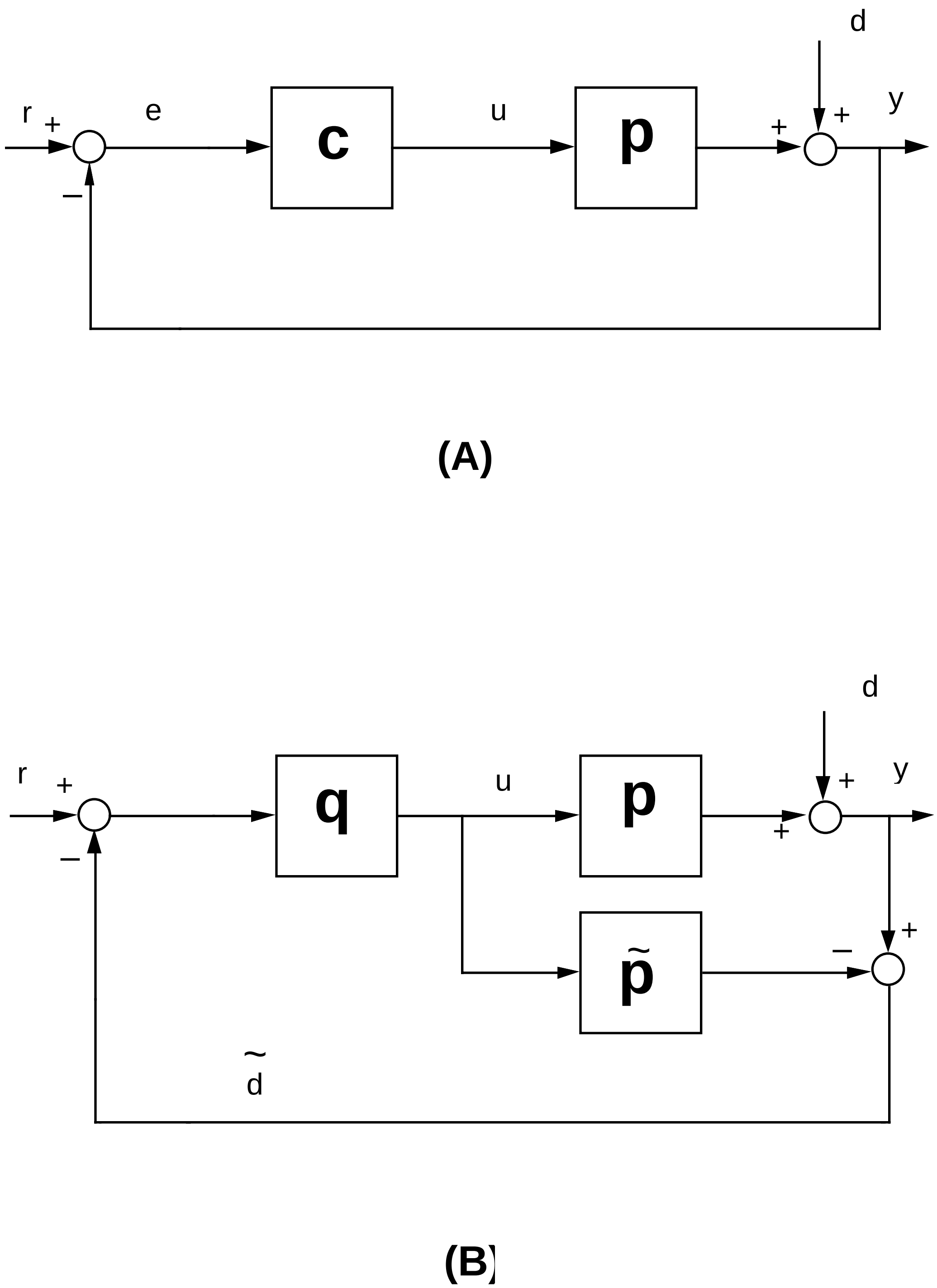

Figure 1: Classical (A) and Internal Model Control (B) Feedback Structures.

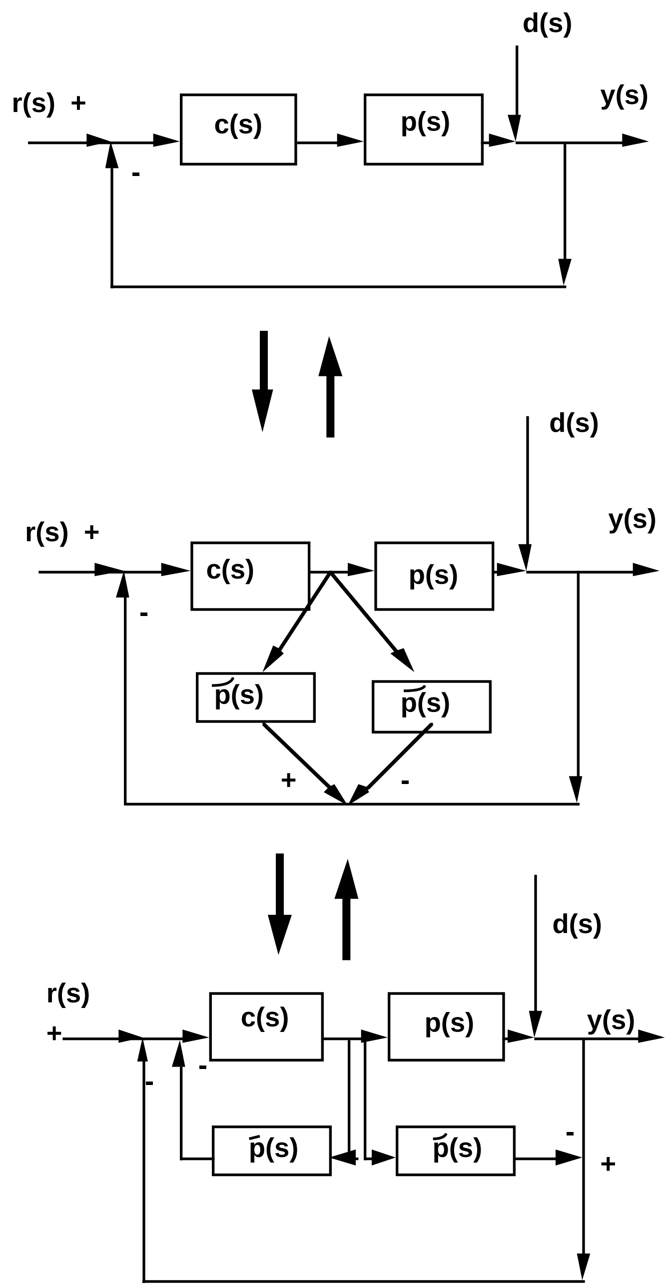

Figure 2: Evolution of the Internal Model Control Feedback Structure.

The IMC structure thus offers the following benefits with respect to classical feedback:

- no need to solve for roots of the characteristic polynomial $1 + pc$; one simply examines the poles of q;
- one can search for q instead of c without any loss of generality.

1.3 Regarding Implementation

For linear, stable plants in the absence of constraints on u, it makes no difference to implement the controller either through c or q. However, in the presence of actuator constraints, one can use the IMC structure to avoid saturation problems without the need for special anti-windup measures.

1.4 Asymptotic closed-loop behavior (System Type)

We need to insure that the feedback control system leads to no offset for setpoint or disturbance changes; we thus need to define so-called Type 1 and Type 2 inputs:

Type 1 (Step Inputs): No offset to asymptotically step setpoint/disturbance changes is obtained if

$$\lim_{s \to 0} \tilde{p} q = \tilde{\eta}(0) = 1$$

Type 2 (Ramp Inputs): For no offset to ramp inputs, it is required that

$$\begin{aligned} \lim_{s \to 0} \tilde{p} q &= \tilde{\eta}(0) = 1 \\ \lim_{s \to 0} \frac{d}{ds}(\tilde{p} q) &= \left.\frac{d\tilde{\eta}}{ds}\right|_{s=0} = 0 \end{aligned}$$

1.5 Requirements for Physical Realizability on q, the IMC Controller

In order for q, the IMC controller, to result in physically realizable manipulated variable responses, it must satisfy the following criteria:

1. *Stability.* The controller must generate bounded responses to bounded inputs; therefore all poles of q must lie in the open Left-Half Plane.
2. *"Properness."* We established in the second lab prep session that differentiation of step inputs by a feedback controller leads to impulse changes in u, which are not physically

realizable. In order to avoid pure differentiation of signals, we must require that $q(s)$ be **proper**, which means that the quantity

$$\lim_{|s|\to\infty} q(s)$$

must be finite. We say $q(s)$ is **strictly proper** if

$$\lim_{|s|\to\infty} |q(s)| = 0$$

A strictly proper transfer function has a denominator order greater than the numerator order. $q(s)$ is **semi-proper**, that is,

$$\lim_{|s|\to\infty} |q(s)| > 0$$

if the denominator order is equal to the numerator order.

A system that is not strictly proper or semiproper is called **improper**.

3. *Causality.* $q(s)$ must be causal, which means that the controller must not require prediction, i.e., it must rely on *current* and *previous* plant measurements. A simple example of a noncausal transfer function is the *inverse* of a time delay transfer function

$$q(s) = \frac{u(s)}{e(s)} = K_c e^{+\theta s} \tag{9}$$

The inverse transform of (9) relies on *future* inputs to generate a *current* output; it is clearly not realizable:

$$u(t) = K_c e(t + \theta) \tag{10}$$

2 Internal Model Control Design Procedure

The IMC design procedure is a two-step approach that, although sub-optimal in a general (norm) sense, provides a reasonable tradeoff between performance and robustness. The main benefit of the IMC approach is the ability to directly specify the complementary sensitivity and sensitivity functions η and ϵ, which as noted previously, directly specify the nature of the closed-loop response.

2.1 Statement of the IMC Design Procedure

The IMC design procedure consists of two main steps. The first step will insure that q is stable and causal; the second step will require q to be proper.

Step1: Factor the model $\tilde{p}$ into two parts:

$$\tilde{p} = \tilde{p}_+ \tilde{p}_- \tag{11}$$

$\tilde{p}_+$ contains all **Nonminimum Phase Elements** in the plant model, that is all Right-Half-Plane (RHP) zeros and time delays. The factor $\tilde{p}_-$, meanwhile, is **Minimum Phase** and invertible; an IMC controller defined as

$$\tilde{q} = \tilde{p}_-^{-1}$$

is *stable* and *causal*.

The factorization of $\tilde{p}_+$ from $\tilde{p}$ is dependent upon the **objective function** chosen. For example,

$$\tilde{p}_+ = e^{-\theta s} \prod_i (-\beta_i s + 1) \qquad \text{Re}(\beta_i) > 0 \tag{12}$$

is Integral-Absolute-Error (IAE)-optimal for step setpoint and output disturbance changes. Meanwhile, the factorization

$$\tilde{p}_+ = e^{-\theta s} \prod_i \frac{(-\beta_i s + 1)}{(\beta_i s + 1)} \qquad \text{Re}(\beta_i) > 0 \tag{13}$$

is Integral-Square-Error (ISE)-optimal for step setpoint/output disturbance changes. As noted in Morari and Zafiriou [2] using ramp, exponential, or other inputs would imply different factorizations.

Step 2: Augment q with a filter $f(s)$ such that the final IMC controller $q = \tilde{q}f(s)$ is now, in addition to stable and causal, *proper*. With the inclusion of the filter transfer function, the final form for the closed-loop transfer functions characterizing the system is

$$\tilde{\eta} = \tilde{p}\tilde{q}f \tag{14}$$

$$\tilde{\epsilon} = 1 - \tilde{p}\tilde{q}f \tag{15}$$

The inclusion of the filter transfer function in Step 2 means that we no longer obtain "optimal control," as implied in Step 1. We wish to define filter forms that allow for no offset to Type 1 and Type 2 inputs; for no offset to step inputs (Type 1), we must require that $\tilde{\eta}(0) = 1$, which requires that $q(0) = \tilde{p}^{-1}(0)$ and forces

$$f(0) = 1 \tag{16}$$

A common filter choice that conforms to this requirement is

$$f(s) = \frac{1}{(\lambda s + 1)^n} \tag{17}$$

The filter order n is selected large enough to make q proper, while λ is an *adjustable parameter* which determines the speed-of-response. Increasing λ increases the closed-loop time constant and slows the speed of response; decreasing λ does the opposite. λ can be be adjusted on-line to compensate for plant/model mismatch in the design of the control system; the higher the value of λ, the higher the robustness the control system.

For no offset to Type-2 (ramp) inputs, in addition to the requirement (16), the closed-loop system must satisfy the following

$$\frac{d}{ds}(\tilde{p}q)\mid_{s=0} = \left.\frac{d\tilde{\eta}}{ds}\right|_{s=0} = 0 \tag{18}$$

By substituting the expression for q obtained from the two-step IMC design procedure, we can write (18) specifically as

$$\frac{d}{ds}(\tilde{p}_+ f)|_{s=0} = 0 \tag{19}$$

One such filter transfer function which meets the condition (18) is

$$f(s) = \frac{(2\lambda - \tilde{p}_+'(0))s + 1}{(\lambda s + 1)^2} \tag{20}$$

Specific forms for $\tilde{p}_+'(0)$ for various simple factorizations of nonminimum phase elements are shown below:

$$\frac{d}{ds}(e^{-\theta s})|_{s=0} = -\theta \tag{21}$$

$$\frac{d}{ds}(-\beta s + 1)|_{s=0} = -\beta \tag{22}$$

$$\frac{d}{ds}(\frac{-\beta s + 1}{\beta s + 1})|_{s=0} = -2\beta \tag{23}$$

Equation (20) will enable us to obtain PID rules for plants with integrator, as will be shown later in this document.

2.2 Why factor $\tilde{p}$?

Recall that for classical feedback

$$y = \eta r + \epsilon d \tag{24}$$

$$\eta = (1 + pc)^{-1} pc \tag{25}$$

$$\epsilon = (1 + pc)^{-1} \tag{26}$$

Using the IMC structure, for no plant/model mistmatch ($p = \tilde{p}$), we have

$$\tilde{\eta} = \tilde{p}q \qquad \tilde{\epsilon} = 1 - \tilde{p}q$$

"Perfect" control (meaning $y = r$ for all time) is achieved when $\tilde{\eta} = 1$ and $\tilde{\epsilon} = 0$, which implies that

$$q = \tilde{p}^{-1} \tag{27}$$

However, in order for $u = q(r - d)$, the manipulated variable response, to be physically realizable, q must be *stable*, *proper*, and *causal*. Nominimum phase behavior (deadtime and RHP zeros) will cause $q = \tilde{p}^{-1}$ to be noncausal and unstable, respectively; if $\tilde{p}$ is strictly proper, then q will be improper as well. Hence the need for factorization.

One can better understand this discussion by examining a simple example. Consider the plant model

$$\tilde{p}(s) = \frac{K(-\beta s + 1)e^{-\theta s}}{\tau^2 s^2 + 2\xi\tau s + 1} \tag{28}$$

where $\beta > 0$, which implies the presence of a Right-Half Plane zero. Nonminimum phase elemets for this plant are $(e^{-\theta s}(-\beta s + 1)$. The "perfect" IMC controller for this system corrresponds to

$$q = \tilde{p}^{-1} = \frac{\tau^2 s^2 + 2\xi\tau s + 1}{K(-\beta s + 1)} e^{+\theta s}$$

While $y = r$ using this controller, the manipulated variable response is physically unrealizable for two reasons. First, q is unstable as a result of a Right-Half Plane pole arising from $(-\beta s + 1)$. Secondly, q is noncausal because of the presence of the time *lead* term $e^{+\theta s}$.

Applying an appropriate factorization to this model as described earlier results in stable, causal control action; a correctly chosen filter order will insure properness and a physically realizable response. One must keep in mind that the nonminimum phase elements $e^{-\theta s}(-\beta s + 1)$ will always form part of the closed-loop response!

3 Application of IMC Design to PID controller tuning

The IMC control design procedure, when applied to low-order models, will often result in PID and PID-like controllers. Developing these is the focus of this section:

3.1 Example 1: PI Control

A PI tuning rule arises from applying IMC to the first-order model:

$$\tilde{p} = \frac{K}{\tau s + 1} \qquad \tau > 0 \tag{29}$$

under the condition that d and r are step input changes.

Step 1: Factor and invert $\tilde{p}$; since $\tilde{p}_+ = 1$, we obtain:

$$\tilde{q} = \frac{\tau s + 1}{K}$$

Step 2: Augment with a first-order filter

$$f = \frac{1}{(\lambda s + 1)}$$

The final form for q is

$$q = \frac{\tau s + 1}{K(\lambda s + 1)} \tag{30}$$

We can now solve for the classical feedback controller equivalent $c(s)$ to obtain

$$c = \frac{q}{1 - pq} = \frac{\tau}{K\lambda}(1 + \frac{1}{\tau s}) \tag{31}$$

which leads to the tuning rule for a PI controller

$$K_c = \frac{\tau}{K\lambda} \tag{32}$$

$$\tau_I = \tau \tag{33}$$

The corresponding *nominal* closed-loop transfer functions for this control system are

$$\tilde{\eta} = \frac{1}{\lambda s + 1} \qquad \tilde{p}^{-1}\eta = \frac{\tau s + 1}{k(\lambda s + 1)} \qquad \tilde{\epsilon} = \frac{\lambda s}{\lambda s + 1} \tag{34}$$

3.2 Example 1b: PI Control

Consider now the first-order model with Right Half Plane (RHP) zero:

$$\tilde{p}(s) = \frac{K(-\beta s + 1)}{(\tau s + 1)} \qquad \beta, \tau > 0 \tag{35}$$

again under the assumption that the inputs to r and d are steps.

Step 1: Use the IAE-optimal factorization for step inputs:

$$\tilde{p}_+ = (-\beta s + 1) \qquad \tilde{p}_- = \frac{K}{(\tau s + 1)} \qquad \tilde{q} = \frac{(\tau s + 1)}{K} \tag{36}$$

Step 2: Use a first-order filter

$$f = \frac{1}{(\lambda s + 1)} \qquad q = \frac{(\tau s + 1)}{K(\lambda s + 1)} \tag{37}$$

Solving for the classical feedback controller leads to another tuning rule for a PI controller:

$$c(s) = K_c(1 + \frac{1}{\tau_I s}) \tag{38}$$

$$K_c = \frac{\tau}{K(\beta + \lambda)} \qquad \tau_I = \tau$$

3.3 Example 1c: PI with filter control

Consider now the first-order model with Left Half-Plane (LHP) zero:

$$\tilde{p}(s) = \frac{K(\beta s + 1)}{(\tau s + 1)} \qquad \beta > 0 \; \tau > 0 \tag{39}$$

again under the assumption that the inputs to r and d are steps.

Step 1: No nonminimum phase behavior in $\tilde{p}$; since $\tilde{p}_+ = 1$, we obtain:

$$\tilde{p}_- = \frac{K(\beta s+1)}{(\tau s+1)} \qquad \tilde{q} = \frac{(\tau s+1)}{K(\beta s+1)} \tag{40}$$

Step 2: Use a first-order filter (q is now *strictly proper*).

$$f = \frac{1}{(\lambda s+1)} \qquad q = \frac{(\tau s+1)}{K(\beta s+1)(\lambda s+1)} \tag{41}$$

Solving for the classical feedback controller $c = \frac{q}{1-\tilde{p}q}$ leads to a tuning rule for an PI *with filter* controller:

$$c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)\frac{1}{(\tau_F s+1)} \tag{42}$$

$$\begin{aligned} K_c &= \frac{\tau}{K\lambda} \\ \tau_I &= \tau \\ \tau_F &= \beta \end{aligned}$$

It is interesting to note that in IMC design, the presence of a Left-Half Plane zero in the model leads a low-pass filter element in the classical feedback controller!

3.4 Example 2: PID Control

Consider now the second-order model with RHP zero:

$$\tilde{p}(s) = \frac{K(-\beta s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \qquad \beta, \tau_1, \tau_2 > 0$$

again under the assumption that the inputs to r and d are steps.

Step 1: Use the IAE-optimal factorization for step inputs:

$$\tilde{p}_+ = (-\beta s+1) \qquad \tilde{p}_- = \frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \tag{43}$$

$$\tilde{q} = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K} \tag{44}$$

Step 2: Use a first-order filter (even though this means that q will still be improper).

$$f = \frac{1}{(\lambda s+1)} \tag{45}$$

$$q = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K(\lambda s+1)} \tag{46}$$

Solving for the classical feedback controller $c = \frac{q}{1-\tilde{p}q}$ leads to a tuning rule for an ideal PID controller:

$$c(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s) \tag{47}$$

$$K_c = \frac{\tau_1 + \tau_2}{K(\beta + \lambda)} \tag{48}$$

$$\tau_I = \tau_1 + \tau_2 \tag{49}$$

$$\tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \tag{50}$$

3.5 Example 3: PID with Filter Control

Consider a second-order model with RHP zero

$$\tilde{p}(s) = \frac{K(-\beta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad \beta, \tau_1, \tau_2 > 0 \tag{51}$$

$\beta > 0$, as before, and subject to step inputs to the closed-loop system. Applying the IMC design procedure gives:

Step 1: Use the ISE-optimal factorization

$$\tilde{p}_+ = \frac{-\beta s + 1}{\beta s + 1} \qquad \tilde{p}_- = \frac{K(\beta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{52}$$

Step 2: A first-order filter leads to q which is semiproper:

$$q = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K(\beta s + 1)(\lambda s + 1)} \qquad f = \frac{1}{\lambda s + 1} \tag{53}$$

Solving for $c(s)$ as before results in a *filtered* ideal PID controller

$$c = K_c(1 + \frac{1}{\tau_I s} + \tau_D s)\frac{1}{(\tau_F s + 1)}$$

with the associated tuning rule

$$K_c = \frac{(\tau_1 + \tau_2)}{K(2\beta + \lambda)} \tag{54}$$

$$\tau_I = \tau_1 + \tau_2 \tag{55}$$

$$\tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \tag{56}$$

$$\tau_F = \frac{\beta \lambda}{2\beta + \lambda} \tag{57}$$

Note the insight given by IMC design procedure regarding on-line adjustment (by changing the value for the IMC filter parameter λ).

3.6 Example 4: Deadtime compensation (PI controller + Smith Predictor)

Consider the first-order with delay plant

$$\tilde{p}(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$$

and step setpoint/output disturbance changes to the closed-loop system.

Step 1: The optimal factorization (IAE, ISE, or otherwise) is $\tilde{p}_+ = e^{-\theta s}$, resulting in:

$$\tilde{q} = \tilde{p}_-^{-1} = \frac{\tau s + 1}{K}$$

Step 2: A first-order filter makes q semiproper;

$$q = \frac{\tau s + 1}{K(\lambda s + 1)} \qquad \tilde{\eta} = \frac{e^{-\theta s}}{(\lambda s + 1)} \tag{58}$$

The corresponding feedback controller is

$$c(s) = \frac{\tau s + 1}{K(\lambda s + 1 - e^{-\theta s})} \tag{59}$$

which can be expressed as a PI controller using the Smith Predictor structure (see Figure 17.4, page 605 in Ogunnaike and Ray).

3.7 PID control for plants with integrator

For plants with integrator, we need to keep in mind that the practical problem will most likely demand no offset for Type-2 inputs, for example, ramp output disturbances ($d = \frac{A}{s^2}$). The application of a Type-2 filter meeting the requirement

$$\frac{d}{ds}(\tilde{p}_+ f) = \frac{d}{ds}(\tilde{\eta})|_{s=0} = 0 \tag{60}$$

as described in Section 2.1 is necessary in order to meet this requirement.

Various cases of PI, PID, and PID with filter controller tuning rules arising from plants with integrator are described in references [1] and [2], and summarized in Table 1; note the progression in controller sophistication as closed-loop performance requirements increase!

4 PID Tuning Rules for 1st-order with Deadtime Plants

A summary of the PI, PID, and PID with filter tuning rules for first-order plants with deadtime is found in Table 3. The PID tuning rule for plants with deadtime arises from

Plant	$\tilde{\eta} = \tilde{p}q = \tilde{p}_+ f$	Controller $c(s)$	No Offset Conditions
$\frac{K(-\beta s+1)}{s}$	$\frac{-\beta s+1}{\lambda s+1}$	P	Steps only
$\frac{K(-\beta s+1)}{s}$	$\frac{(-\beta s+1)}{(\beta s+1)(\lambda s+1)}$	P with filter	Steps Only
$\frac{K(-\beta s+1)}{s}$	$\frac{(-\beta s+1)[(\beta+2\lambda)s+1]}{(\lambda s+1)^2}$	PI	Steps and Ramps
$\frac{K(-\beta s+1)}{s}$	$\frac{(-\beta s+1)}{(\beta s+1)}\frac{[2(\beta+\lambda)s+1]}{(\lambda s+1)^2}$	PI with filter	Steps and Ramps
$\frac{K(-\beta s+1)}{s(\tau s+1)}$	$\frac{(-\beta s+1)}{\beta s+1)}\frac{[2(\beta+\lambda)s+1]}{(\lambda s+1)^2}$	PID with filter	Steps and Ramps

Table 1: PID tuning rules for plants with integrator

using a first-order Padé approximation in lieu of the time delay.

$$p = \frac{Ke^{-\theta s}}{\tau s + 1} \tag{61}$$

$$\approx \frac{K(-\frac{\theta}{2}s+1)}{(\frac{\theta}{2}s+1)(\tau s+1)} \tag{62}$$

The Padé-approximated plant (62) is a second-order plant with RHP zero; using the analysis from the **Example 2: PID Control** subsection leads to a PID tuning rule:

$$K_c = \frac{2\tau + \theta}{K(2\lambda + \theta)} \tag{63}$$

$$\tau_I = \tau + \frac{\theta}{2} \tag{64}$$

$$\tau_D = \frac{\tau\theta}{2\tau + \theta} \tag{65}$$

As shown in Rivera *et al.* [1] the ratio of the ISE objective function for the PID control system

$$J = \text{ ISE } = \int_0^\infty (y - r)^2 dt \tag{66}$$

versus the optimal ISE for a first-order with deadtime plant

$$J_{opt} = \theta^2 \tag{67}$$

can be plotted as a function of $\frac{\lambda}{\theta}$ independent of τ, as noted in Figure 3. Figure 3 also shows M, which represents the maximum peak of the nominal complementary sensitivity function

$$M = \sup_\omega \eta \tag{68}$$

This measure can be related to robustness of the closed-loop system, as described in [1]. Note that at $\frac{\lambda}{\theta} \approx 0.8$ the IMC-PID controller results in an ISE value that is only 10% greater

Table 2: IMC-Based Tuning for Ideal PID Controllers Using Simple Models

$$c(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s)\frac{1}{(\tau_F s + 1)}$$

Model	Input v_M	$\tilde{\eta} = \tilde{p}q = \tilde{p}\tilde{q}f$	K_cK	τ_I	τ_D	τ_F
$\frac{K}{\tau s+1}$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{\tau}{\lambda}$	τ	-	-
$\frac{K}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{2\zeta\tau}{\lambda}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	-
$\frac{K(-\beta s+1)}{\tau s+1}$ $\beta > 0$	$\frac{1}{s}$	$\frac{-\beta s+1}{\lambda s+1}$	$\frac{\tau}{\beta+\lambda}$	τ	-	-
$\frac{K(-\beta s+1)}{\tau s+1}$ $\beta > 0$	$\frac{1}{s}$	$\frac{(-\beta s+1)}{(\beta s+1)(\lambda s+1)}$	$\frac{\tau}{2\beta+\lambda}$	τ	-	$\frac{\beta\lambda}{2\beta+\lambda}$
$\frac{K(-\beta s+1)}{\tau s+1}$ $\beta < 0$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{\tau}{\lambda}$	τ	-	$-\beta$
$\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1}$ $\beta > 0$	$\frac{1}{s}$	$\frac{(-\beta s+1)}{\lambda s+1}$	$\frac{2\zeta\tau}{\beta+\lambda}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	-
$\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1}$ $\beta > 0$	$\frac{1}{s}$	$\frac{(-\beta s+1)}{(\beta s+1)(\lambda s+1)}$	$\frac{2\zeta\tau}{2\beta+\lambda}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	$\frac{\beta\lambda}{2\beta+\lambda}$
$\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1}$ $\beta < 0$	$\frac{1}{s}$	$\frac{1}{\lambda s+1}$	$\frac{2\zeta\tau}{\lambda}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$	$-\beta$
$\frac{K}{s}$	$\frac{1}{s^2}$	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2}{\lambda}$	2λ	-	-
$\frac{K}{s(\tau s+1)}$	$\frac{1}{s^2}$	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2\lambda+\tau}{\lambda^2}$	$2\lambda + \tau$	$\frac{2\lambda\tau}{2\lambda+\tau}$	-
$\frac{K(-\beta s+1)}{s}$ $\beta > 0$	$\frac{1}{s^2}$	$\frac{(-\beta s+1)(2\lambda+\beta)s+1}{(\lambda s+1)^2}$	$\frac{2\lambda+\beta}{(\lambda+\beta)^2}$	$2\lambda + \beta$	-	-
$\frac{K(-\beta s+1)}{s}$ $\beta > 0$	$\frac{1}{s^2}$	$\frac{(-\beta s+1)(2(\beta+\lambda)s+1)}{(\beta s+1)(\lambda s+1)^2}$	$\frac{2(\beta+\lambda)}{2\beta^2+4\beta\lambda+\lambda^2}$	$2(\beta + \lambda)$	-	$\frac{\beta\lambda^2}{2\beta^2+4\beta\lambda+\lambda^2}$
$\frac{K(-\beta s+1)}{s}$ $\beta < 0$	$\frac{1}{s^2}$	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2}{\lambda}$	2λ	-	$-\beta$
$\frac{K(-\beta s+1)}{s(\tau s+1)}$ $\beta > 0$	$\frac{1}{s^2}$	$\frac{(-\beta s+1)((\beta+2\lambda)s+1)}{(\lambda s+1)^2}$	$\frac{\beta+2\lambda+\tau}{(\beta+\lambda)^2}$	$\beta + 2\lambda + \tau$	$\frac{\tau(\beta+2\lambda)}{\beta+2\lambda+\tau}$	-
$\frac{K(-\beta s+1)}{s(\tau s+1)}$ $\beta > 0$	$\frac{1}{s^2}$	$\frac{(-\beta s+1)(2(\beta+\lambda)s+1)}{(\beta s+1)(\lambda s+1)^2}$	$\frac{2(\beta+\lambda)+\tau}{2\beta^2+4\beta\lambda+\lambda^2}$	$2(\beta + \lambda) + \tau$	$\frac{2\tau(\beta+\lambda)}{2(\beta+\lambda)+\tau}$	$\frac{\beta\lambda 2}{2\beta^2+4\beta\lambda+\lambda^2}$
$\frac{K(-\beta s+1)}{s(\tau s+1)}$ $\beta < 0$	$\frac{1}{s^2}$	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2\lambda+\tau}{\lambda^2}$	$2\lambda + \tau$	$\frac{2\tau\lambda}{2\lambda+\tau}$	$-\beta$

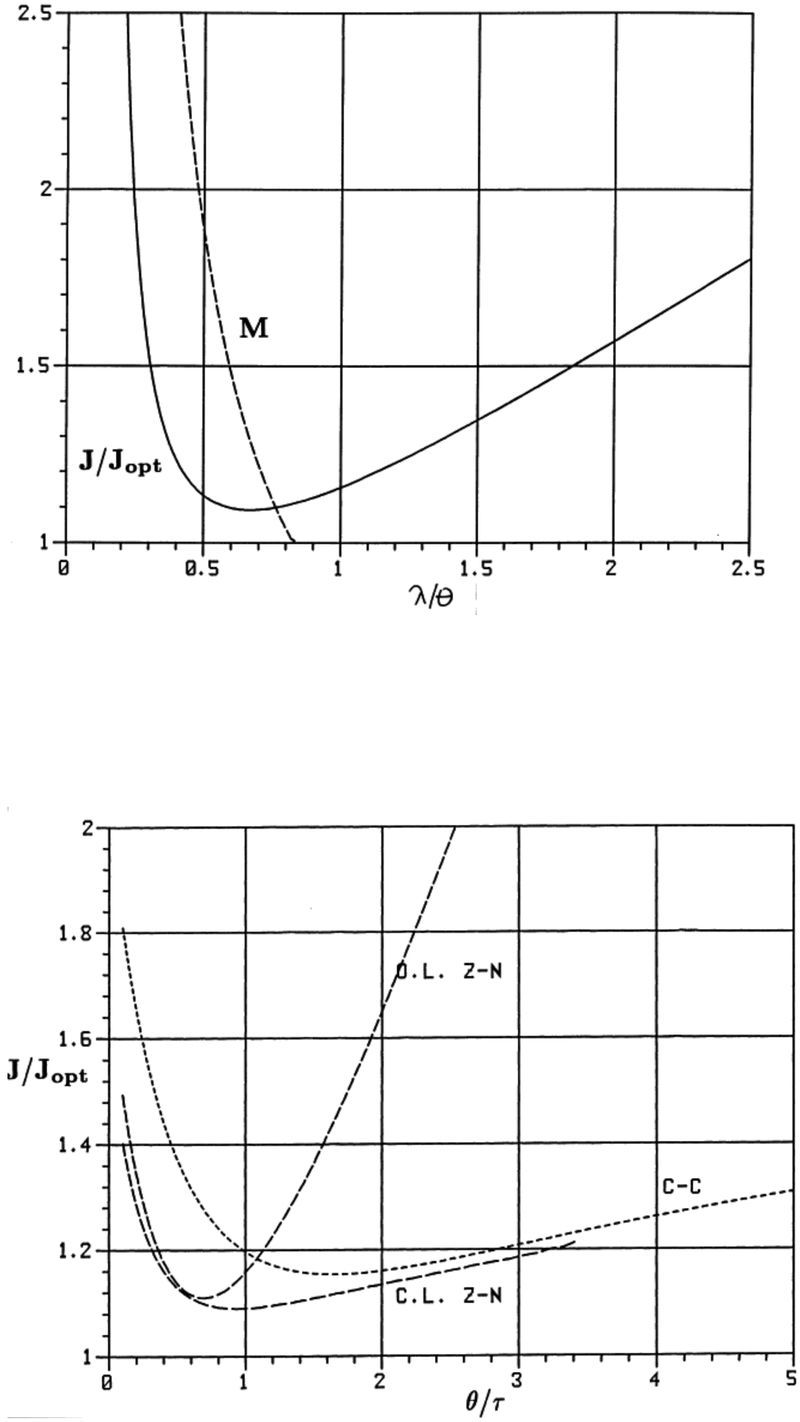

Figure 3: J/J_{opt} and M for the IMC-PID controller (top), and comparison with other methods (bottom): open-loop Ziegler-Nichols (O-L Z-N), closed-loop Ziegler-Nichols (C-L Z-N), and Cohen-Coon (C-C).

Controller	KK_c	τ_I	τ_D	τ_F	Recommended $\frac{\lambda}{\theta}(\lambda > 0.2\tau$ always)
"Original" PI	$\frac{\tau}{\lambda}$	τ	–	–	> 1.7
"Improved" PI	$\frac{2\tau+\theta}{2\lambda}$	$\tau + \frac{\theta}{2}$	–	–	> 1.7
PID	$\frac{2\tau+\theta}{(2\lambda+\theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{(2\tau+\theta)}$	–	> 0.8
PID with filter	$\frac{2\tau+\theta}{2(\lambda+\theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$	$\frac{\lambda\theta}{2(\lambda+\theta)}$	> 0.25

Table 3: IMC-based tuning rules for PI, PID, and PID with filter controllers for a first-order with deadtime system

than optimal, while maintaining a low value for M. The controlled variable response of the IMC-PID controller for various settings of $\frac{\lambda}{\theta}$ is shown in Figure 4.

The "original" PI tuning rule is found by approximating the first-order delay plant with just the first-order lag term, without delay:

$$p = \frac{Ke^{-\theta s}}{\tau s + 1} \approx \frac{K}{\tau s + 1} \tag{69}$$

Figure 5 shows a marked deterioration in achievable ISE performance, relative to the PID tuning rule. At its best setting ($\frac{\lambda}{\theta} \approx 1.35$) the IMC-PI controller results in an ISE value that is over 50% greater than optimal, with a high value for the complementary sensitivity function, $M \approx 1.4$. The "Improved" PI rule arises by incorporating the delay in the time constant of the internal model $\tilde{p}$

$$p = \frac{Ke^{-\theta s}}{\tau s + 1} \approx \frac{K}{(\tau + \frac{\theta}{2})s + 1} \tag{70}$$

resulting, as shown in **Example 1: PI control** in the tuning rule:

$$K_c = \frac{2\tau + \theta}{2K\lambda} \tag{71}$$

$$\tau_I = \tau + \frac{\theta}{2} \tag{72}$$

The improved PI rules, as the name implies, result in superior performance over the standard IMC-PI rules; however, the performance obtained from these rules varies as a function of θ/τ. A "worst-case" performance and robustness analysis with respect to λ/θ for a wide range of θ/τ is presented in Figure 6 (top). Evaluating the improved PI tuning rule for a specific choice of $\lambda/\theta = 1.7$ shows that the corresponding performance is superior to that of the Cohen-Coon and closed-loop Ziegler-Nichols rules over most of the θ/τ range, as noted in Figure 6 (bottom).

Tuning rules for a PID with filter controller (shown in Table 3) can be obtained as well using (62) and the analysis of **Example 3: PID with Filter Control**, leading to the result

$$K_c = \frac{2\tau + \theta}{2K(\lambda + \theta)} \tag{73}$$

$$\tau_I = \tau + \frac{\theta}{2} \tag{74}$$

$$\tau_D = \frac{\tau\theta}{2\tau + \theta} \tag{75}$$

$$\tau_F = \frac{\lambda\theta}{2(\lambda + \theta)} \tag{76}$$

Figure 7 shows the ISE performance obtained from the PID with filter tuning rule. Comparing Figure 7 with Figure 3, one notices that the IMC-PID with filter tuning leads to higher ISE than the IMC-PID for the same value of λ/θ; however, the PID with filter settings display much smoother closed-loop responses, as evidenced in Figure 7 (bottom). In industrial practice, the smoothness of the response may well be worth the loss of performance in terms of ISE.

References

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[2] Morari, M. and E. Zafiriou. **Robust Process Control**, Prentice-Hall, Englewood Cliffs, NJ, 1989.

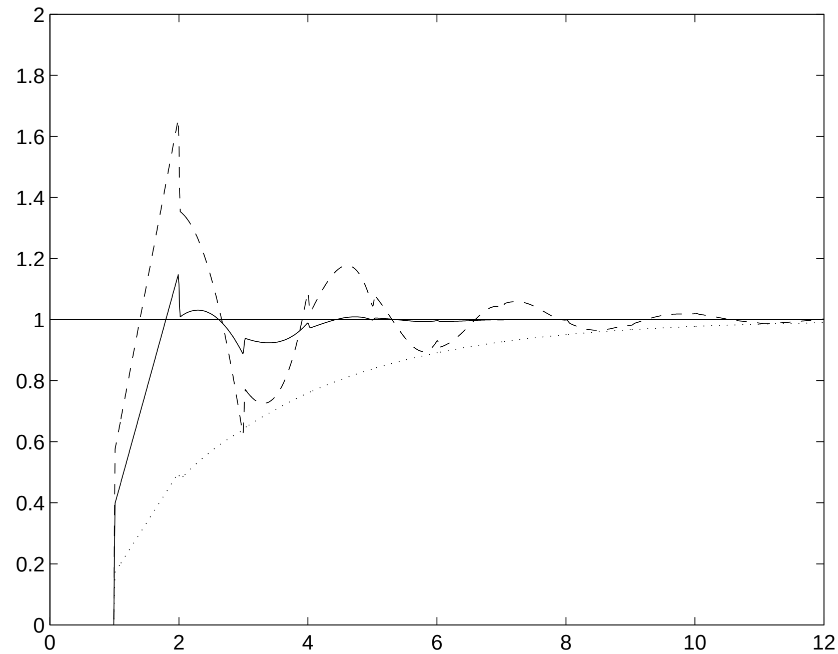


Figure 4: IMC-PID controlled variable responses for a step setpoint change, for various settings of $\frac{\lambda}{\theta}$; solid: $\frac{\lambda}{\theta} = 0.8$; dotted: $\frac{\lambda}{\theta} = 2.5$; dashed: $\frac{\lambda}{\theta} = 0.4$.

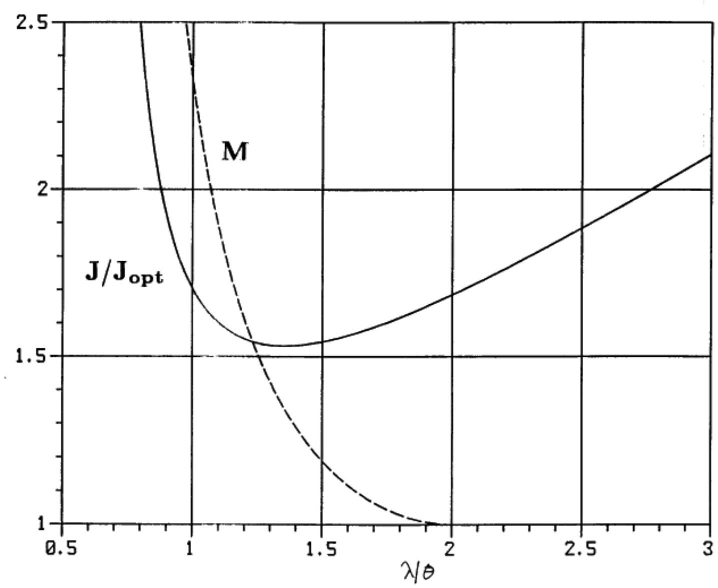


Figure 5: J/J_{opt} and M for the "original" IMC-PI controller.

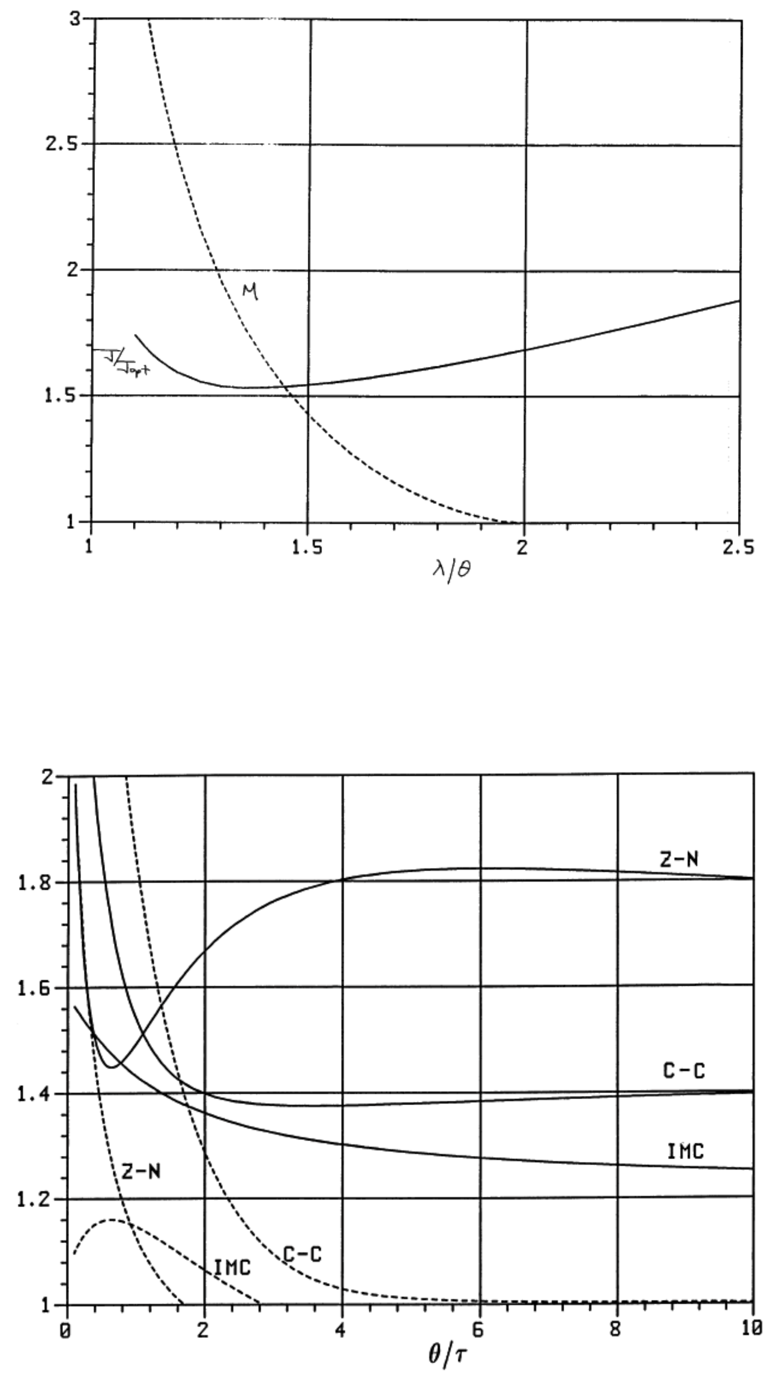
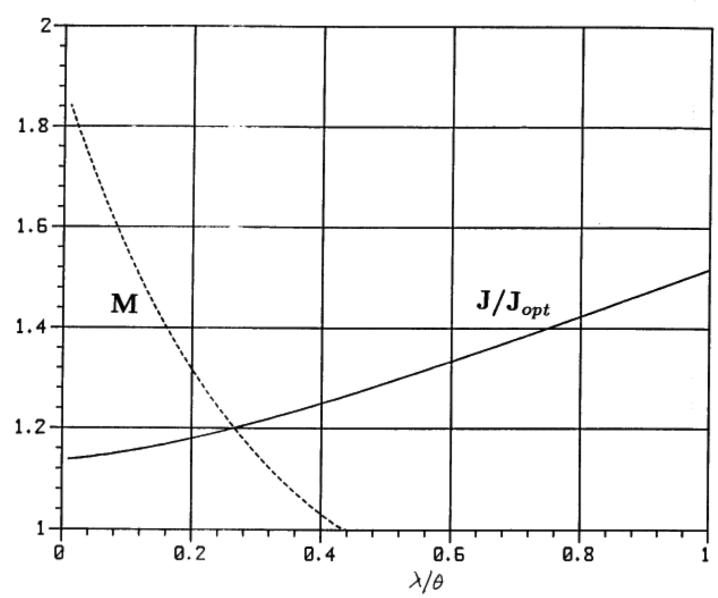


Figure 6: Worst-case J/J_{opt} and M for the "improved" IMC-PI controller (top), and comparison (for $\lambda/\theta = 1.7$) with other methods: closed-loop Ziegler-Nichols (Z-N), and Cohen-Coon (C-C) (bottom). Solid: J/J_{opt}; Dashed: M.



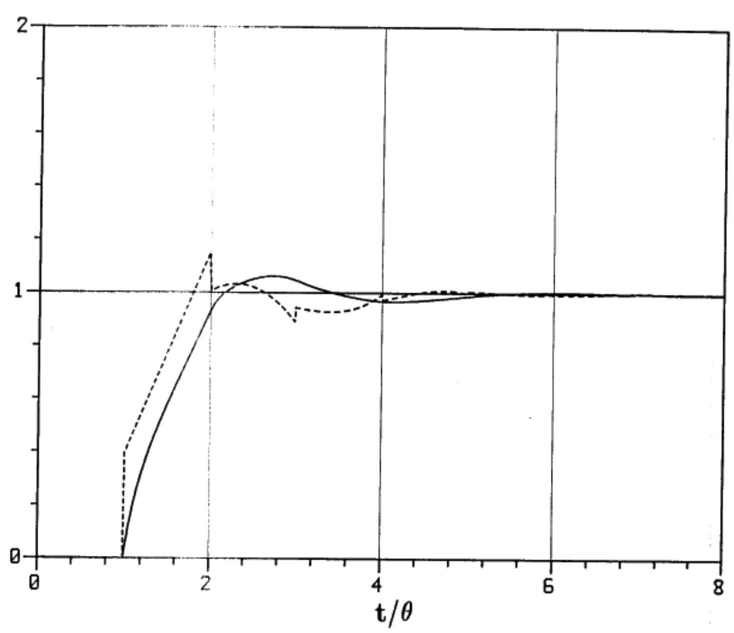


Figure 7: J/J_{opt} and M for the IMC-PID with filter controller (top), and controlled variable response comparison with the IMC-PID rule (bottom). For bottom figure, solid: IMC-PID with filter ($\frac{\lambda}{\theta} = 0.45$); dotted: IMC-PID ($\frac{\lambda}{\theta} = 0.8$);